LOW-FREQUENCY PLASMA OSCILLATIONS IN JUNCTION DIODES

G. V. Gordeev

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The author considers low-frequency plasma oscillations in junction diodes using the kinetic-equation method and allowing for asymmetry in the boundary conditions at places where the plasma adjoins the preelectrode barriers. It is shown that waves moving from the cathode to the anode can be excited at low electron drift velocities than in an unbounded plasma, because these waves are reinforced by waves reflected from the electrodes. The condition for the appearance of nonattenuating plasma oscillations is derived for a wavelength equal to the distance between electrodes.

In [1], the dispersion equation for plasma oscillations in junction diodes was derived using the kinetic-equation method and allowing for asymmetry in the boundary conditions at the plasma-preelectrode boundaries. Asymmetry in the boundary conditions is associated with the presence of an external voltage applied between the electrodes; here the same charge particles can be accelerated as they move to one electrode and decelerated as they move to the other. In the same article [1], the dispersion equation obtained in this manner was studied for the case of high-frequency (electron) plasma oscillations. In this article we shall use the dispersion equation obtained in [1] for low-frequency (ion) plasma oscillations.

In [2], low-frequency oscillations in an unbounded plasma were considered; the condition for wave excitation by the energy of electrons drifting in an external field without the presence of an additional beam of particles was derived. Experimental verification of this theory [3] qualitatively confirmed the theoretical conclusion regarding the excitation of plasma oscillations without the presence of an additional beam of particles. The qualitative results differ somewhat from the experimental results. In this article it will show that the discrepancy between the quantitative theoretical conclsuions and the experimental results is greatly reduced if we allow for the effect of the plasma boundaries on the oscillations.

1. The dispersion equation for plasma oscillations in junction diodes has the form [1]

$$\begin{gathered}\operatorname{Det} |[1 + A_1(k) + A_2(k)]\,\delta_{kk_1} + \\ + B_1(-k, -k_1) + B_2(k, k_1)\,| = 0\,. \end{gathered} \tag{1.1}$$

Here $k = \pi n/l$ and $k_1 = \pi m/l$ are wave numbers ($k > 0$ if the wave travels from the anode to the cathode and $k < 0$ if it moves in the opposite direction); l is one half the distance between the cathode and anode plasma boundaries; n and m are integers. The quantities A_α and B_α are expressed in terms of very complex integrals whose form is given in [1]. We only note that B_α is the correction to A_α that results when we allow for the effect of boundaries on the plasma oscillations. Consider low-frequency oscillations whose attenuation decrement γ is mcuh less than their frequency ω. The phase velocity of such oscillations falls within a narrow interval: it muct be mcuh less than the electron thermal (isothermal) velocity [2]; it can be one or (in extreme cases) two orders less than the thermal velocity.

For these waves, approximate calculation of the terms B_2 and A_α yields

$$A_1 = \frac{1}{k^2 a_1^2} + i\left(\frac{\pi}{2}\right)^{1/2} \frac{[\omega - (\omega k)]}{k^2 a_1^2 s_1 |k|}, \qquad A_2 = -\frac{\omega_2^2}{[\omega - i(\gamma - \tau_2^{-1})]^2}, \tag{1.2}$$

$$B_2(k, k_1) = (-1)^{n-m} \frac{i\omega_2^2}{\omega^2 kl}\left(1 + \frac{2}{\sqrt{2\pi}}\frac{w_2}{s_2}\right),$$

$$a_1 = \left(\frac{\varkappa T_1}{4\pi e^2 n_1}\right)^{1/2}, \qquad s_\alpha = \left(\frac{\varkappa T_\alpha}{m_\alpha}\right)^{1/2}, \qquad \omega_2 = \left(\frac{4\pi e^2 n_2}{m_2}\right)^{1/2}.$$

Here ω is the oscillation frequency; γ is the attenuation decrement; a_1 is the electron Debye radius; ηT_1 is the electron temperature in ergs; n_1 is the electron density; e is the charge on an electron; w_α is the drift velocity of electrons ($\alpha = 1$) and ion ($\alpha = 2$); s_α is their thermal (isothermal) velocity; w_2 is the limit frequency of plasma-ion oscillations; n_2 is the ion density; m_2 is the ion mass; τ_2 is the time between ion collision with neutral atoms. It is somewhat difficult to calculate B_1 since for $|\beta_2| \ll 1$ the stationary-phase method used in estimating the omitted terms in B_2 is not always applicable. Let n be the number of waves adding up to the distance between electrodes. We set $\delta = 2\pi n\omega/ks_1$.

If $\delta \gg 1$ we can use the stationary-phase method and, by calculating B_1 approximately, we have

$$B_1(-k, -k_1) = (-1)^{n-m} \frac{i}{4k^2 a_1^2 k_1 l}. \tag{1.3}$$

If, however, $\delta < 1$, it is not convenient to use the stationary-phase method to evaluate the integrals in the expression for B_1; another method is therefore necessary.

If $\delta \ll 1$, it is more convenient to integrate from 0 and 1 and from 1 to ∞ in integrals of type

$$J_\alpha = \frac{1}{\sqrt{2\pi}} \int^{\infty} \exp \varphi_\alpha(u)\, du \qquad \left(\varphi_\alpha = \frac{i\alpha\delta}{u} - \frac{u^2}{2}, \quad \alpha = 1.2\right)$$

which appear in the expression for B_1 (see Eq. (20) of [1]); we have now the approximate expression

$$J_\alpha = \frac{1}{\sqrt{2\pi}} \int_0^1 e^{i\alpha\delta/u}\, du + \frac{1}{\sqrt{2\pi}} \int_1^\infty e^{-\frac{1}{2}u^2}\, du \approx \frac{1}{\sqrt{2\pi}} e^{-i\alpha\delta} + 0.16.$$

Thus, for $\delta \ll 1$, we have the following approximate expression for B_1:

$$B_1(-k, -k_1) \approx (-1)^{n-m} \frac{i}{4k^2 a_1^2 k_1 l}\left[2.28 + \frac{4}{\sqrt{2\pi}}(e^{-i\delta} + e^{-i2\delta})\right] \approx$$
$$\approx (-1)^{n-m} \frac{i}{4k^2 a_1^2 k_1 l}\left(5.5 - \frac{12}{\sqrt{2\pi}} i \sin\delta\right). \tag{1.4}$$

Substituting (1.2) and (1.3) or (1.4) (depending on whether $\delta \gg 1$ or $\delta \ll 1$) into Eq. (1.1), isolating the real and imaginary parts of Eq. (1.1), and solving for the wave attenuation and frequency, we can obtain corrections due to the presence of plasma boundaries.

2. Let us first study the case in which $\delta \gg 1$. In this case, the correction terms B_α do not affect the oscillation frequency. For this frequency, the familiar Langmuir-Tonks formula is obtained [4]:

$$\omega = \omega_2 a_1 k / \sqrt{1 + k^2 a_1^2} + (\mathbf{w}_2 \cdot \mathbf{k}). \tag{2.1}$$

For attenuation, we have the approximate expression

$$\gamma = \frac{1}{\tau_2} + \left(\frac{\pi}{2}\right)^{1/2} \frac{[\omega - (\mathbf{w}_1\mathbf{k})]\,\omega_2}{(1 + k^2 a_1^2)^{3/2}\,\omega_1} + \frac{|k|\, a_1 \omega_2}{4(1 + k^2 a_1^2)^{3/2}\, kl} \times$$
$$\times \left[1 + (1 + k^2 a_1^2)\left(1 + \frac{2}{\sqrt{2\pi}} \frac{w_2}{s_2}\right)\right], \qquad \omega_1 = (4\pi e^2 n_1 / m_1)^{1/2}. \tag{2.2}$$

Here ω is the Langmuir frequency of plasma-electron oscillations. The first two terms on the right-hand side of (2.2) determine wave attenuation in an unbounded plasma [2]; the last term is the correction resulting from the effect of boundaries on the plasma oscillations. As is shown in [2], low-frequency wave propagating in the direction of electron drift ($k < 0$) can be excited by the energy acquired by an electron in an external field. The wave number k_* corresponding to a maximum oscillation increment is defined (in an unbounded plasma) by the relation

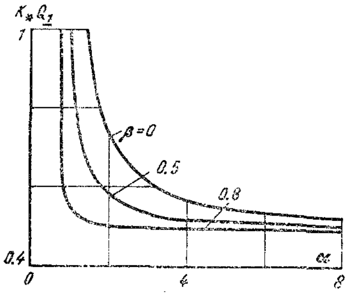
$$k_*a_1 = \frac{\omega a_2}{u_1}\left(1 + \frac{k_*^2 a_1^2}{2k_*^2 a_1^2 - 1}\right). \tag{2.3}$$

This equation is valid given the condition $2k^2a^2{}_1 - 1 > 0$. The minimum frequency simultaneously satisfying both (2.1) and (2.3) is realized when $k^2a^2 \to 1/2$ and $\omega_1/v_s \to \infty$ where $v_s = (\varkappa T_1/m_2)^{1/2}$ is the ion speed of sound. This quantity depends on the ion drift velocity. If $\omega_2 \ll v_s$, then $\omega_{min} = 0.6\,\omega_2$; if, however, $\omega_2 \to 0.8\,v_s$, then $\omega_{min} \to 0$.

When the effect of boundaries on the plasma oscillations is allowed for, the wave number k_* corresponding to a maximum increment is given by

$$k_*a_1 = \frac{\omega a_1}{w_1}\left(1 + \frac{k_*^2 a_1^2}{2k_*^2 a_1^2 - 1}\right) - \frac{s_1}{w_1}\frac{k_*^2 a_1^2}{k_*^2 a_1^2 - 1}\left[2 + k_*^2 a_1^2 + \frac{2}{\sqrt{2\pi}}\frac{u_2}{s_2}(1 + k_*^2 a_1^2)\right]. \tag{2.4}$$

As is clear from (2.2) and (2.4), for a fixed gas pressure the electron drift velocity must be smaller in magnitude than the distance between electrodes for establishing nonattenuating waves. A decrease in the electron drift velocity required to establish a nonattenuating wave (assuming the presence of plasma boundaries) is associated with the fact that the energy lost by a wave through attenuation is partially compensated for the energy of wave reflected from the electrodes. As the distance between electrodes decreases, k_* shifts toward smaller k (assuming the plasma parameters and drift velocity of the charged particles remain constant). The figure shows graphs of k_*a_1 as a function of the parameter $\alpha = \omega_1/v_s$ for certain values of the parameter $\beta = s_1/\omega_2 l$. It is clear from this figure that the greater the value of β the smaller the value of α required to reach the limit value $k^2a^2{}_1 = 1/2$. In particular for $\beta = 0.8$, $k^2a^2{}_1 \approx 1/2$ even for $\alpha = 2$. If for these values of α and β we set $w_2 = 0.1$ (this is admissible because of the effect of ion charge transfer), then $w = 0.53\omega_2$ which is $0.1\ \omega_2$ smaller than the value of ω obtained for the same values of α and ω_2 but for $\beta = 0$. Therefore, the greater β, the more closely the calculated value for the frequency approaches its experimental value [3]. It is impossible to expect theory and experiment to coincide exactly since the experiment [3] was performed with a plasma in a gas discharge tube in which the side boundaries of the plasma were of considerable importance; these factors were not allowed for in our theory.



Graph of k_*a_1 as a function of α for different β.

3. Now consider the case $\delta \ll 1$. Since the wave phase velocity is one or two orders less than the electron thermal velocity, the case $\delta \ll 1$ can only occur when $n = 1$. We shall consider the limit case $n = 1$ or $k = k_1$ where

$$k_1 l = \pi. \tag{3.1}$$

The width of the preelectrode layers is on the order of the electron Debye radius. Equation (1.1) is derived assuming that the width of the preelectrode layers is much less than the distance between electrodes or $a_1 \ll 1$. It now follows from (3.1) that

$$k_1 a_1 \ll \pi. \tag{3.2}$$

The correction to B_1 must now be found using (1.4) rather than (1.3). The only effect on the oscillation frequency is that in this case there is an additional term $3\delta/\sqrt{mkl}$ in (2.1). Allowing for (3.1) and (2.2) we obtain

$$\omega = \frac{\pi a_1 \omega_2}{l\sqrt{1 + \pi^2 a_1^2/l^2 + 6\omega_2/\sqrt{2\pi}\,\omega_1}}. \tag{3.3}$$

Since $\omega_2/\omega_1 = \sqrt{m/M} \ll 1$, the correction $6\omega_2/2\pi\omega_1$ to the 1 in the square root of (3.3) will be insignificant; this means that the correction to B_1 will not noticeably affect the oscillation frequency for $\delta \ll 1$. Ignoring this correction and the correction $\pi^2 a^2{}_1/l^2$ to the 1 in the square root in (3.3), we obtain the approximate expression

$$\omega = \pi a_1 \omega_2 / l, \tag{3.4}$$

where the wave phase velocity is equal to the ion sound velocity. Oscillations with a frequency inversely proportional to the distance between electrodes were observed in a thermionic converter [5]. Consider the condition under which oscillations, having the frequency of (3.4) become nonattenuating. For $\delta \ll 1$ wave attenuation is determined by the same formula (2.2), only the number 5.5 must be substituted for the 1 in the square root. Letting $\gamma = 0$ and using (3.1) and (3.2) we obtain the condition for establishing nonattenuating oscillations with $k = \pi/l$ (for $\omega_2 \ll s_2$):

$$l = l_2 \sqrt{T_1/T_2}\,(1.6 + \pi \sqrt{1/2\pi}\, w_1/s_1),$$

where l_2 is the ion mean free path and T_1 and T_2 are the electron and ion temperatures. As is clear from (3.5), nonattenuating plasma oscillations with wave number $k = \pi/l$ and frequency (3.4) are possible when the distance between electrodes is equal to a multiple of the ion mean free path.

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